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MASS TRANSFER TO THE REAR OF A SPHERE IN STOKES FLOW

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Abstract—Mass transfer to a sphere, including the rear region, in Stokes flow and at large Péclet numbers, is investigated. By the singular-perturbation technique, six distinct regions of different mass-transfer mechanisms are found. One of these regions, the diffusion layer, has already been solved by the boundary-layer method. Another area, the region at the rear of the sphere, is solved here. The local Nusselt number at the rear stagnation point is found to be 1·192. To predict the mass transfer rate everywhere on the sphere, a composite solution can be formed from the boundary-layer solution and the rear-region solution.
In heat- and mass-transfer problems, the method used here complements the boundary-layer methods in predicting the heat- or mass-transfer rate at the rear of an axisymmetric object.

NOMENCLATURE

c_i, concentration of the diffusing species;
c_0, concentration at the surface;
c_∞, concentration in the bulk solution;
D, diffusion coefficient;
f, asymptotic solution for $\epsilon\Theta/Y^{\frac{1}{2}}$ as $Y \to \infty$ in region 5;
g, see equation (4);
Nu, the local Nusselt number;
Pe, $2Rv_\infty/D$, the Péclet number;
r, radial coordinate;
r^*, dimensionless radial coordinate;
R, radius of the sphere;
$\mathcal{R}$, ϵr^*, stretched coordinate for region 6;
s, $= S/\sqrt{\epsilon} = \theta/\epsilon^{\frac{3}{2}}$, stretched coordinate for region 4;
S, θ/ϵ, stretched coordinate for regions 3 and 5;
$\mathcal{S}$, $= \theta/\epsilon^2$, stretched coordinate for region 6;
v_r^*, dimensionless radial velocity;
v_θ^*, dimensionless tangential velocity;
v_∞, velocity far from the sphere;
x, distance measured along the rear axis;
y, $= r^* - 1$, the normal distance from the sphere;
Y, $= y/\epsilon$, a stretched diffusion-layer coordinate.

Greek symbols
$\Gamma(\frac{4}{3})$, $= 0{\cdot}89298$, the gamma function of $\frac{4}{3}$;
ϵ, $= (Pe/2)^{-\frac{1}{3}}$, perturbation parameter;
η, $= Y/g(\theta)$, similarity variable for diffusion layer;
θ, angular coordinate, measured from rear axis;
Θ, $= (c_i - c_0)/(c_\infty - c_0)$, dimensionless concentration;
ξ, $= S\sqrt{Y}$, similarity variable for asymptotic solution in region 5;
Ψ, stream function variable appropriate to region 3, see equation (10).

INTRODUCTION

THE PROBLEM of mass (or heat) transfer to a sphere in Stokes flow has been studied by many people; for example, Levich [1], Acrivos and Taylor [2], and Acrivos and Goddard [3]. However, the boundary-layer type of approach used by these authors inevitably breaks down near the rear of the sphere. Although, as pointed out by Acrivos and Goddard, the rate of mass transfer at the rear contributes little to the total mass-transfer rate, it is still physically important in determining the true nature of the processes involved. The method used here is of dual significance. In one sense, it is important for understanding the mass-transfer process of the entire sphere. In another, more general sense, it may be applied to correct boundary-layer solutions of other axisymmetric bodies.

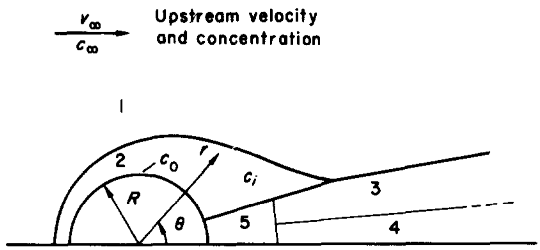

FIG. 1. Regions of different mass-transfer mechanisms for mass transfer to a sphere in Stokes flow at high Péclet numbers.

The particular problem investigated here is shown graphically in Fig. 1. We shall assume throughout this paper that the velocity field around the sphere is described by Stokes' formula. Due to the concentration difference between the sphere and the surrounding fluid, mass transfer will take place. The convective diffusion equation in spherical coordinates can be written as

$$\frac{Pe}{2}\left[v_r^*\frac{\partial\Theta}{\partial r^*}+\frac{v_\theta^*}{r^*}\frac{\partial\Theta}{\partial\theta}\right]=\frac{\partial^2\Theta}{\partial r^{*2}}+\frac{2}{r^*}\frac{\partial\Theta}{\partial r^*}+\frac{1}{r^{*2}}\left(\frac{\partial^2\Theta}{\partial\theta^2}+\frac{\cos\theta}{\sin\theta}\frac{\partial\Theta}{\partial\theta}\right), \qquad (1)$$

where $Pe = 2Rv_\infty/D$, the Péclet number; $r^* = r/R$; $\Theta = (c_i - c_0)/(c_\infty - c_0)$; and

$$v_r^* = \left(1-\frac{3}{2r^*}+\frac{1}{2r^{*3}}\right)\cos\theta;$$

$$v_\theta^* = -\left(1-\frac{3}{4r^*}-\frac{1}{4r^{*3}}\right)\sin\theta.$$

The boundary conditions for equation (1) are:

1. $\Theta = 0$ at $r^* = 1$.
2. $\Theta = 1$ at $r^* = \infty$.
3. $\partial\Theta/\partial\theta = 0$ at $\theta = \pi$, the front axis.
4. $\partial\Theta/\partial\theta = 0$ at $\theta = 0$, the rear axis.

The last two boundary conditions are necessary due to symmetry.

REGIONS OF DIFFERENT MASS-TRANSFER MECHANISMS

In liquid systems where the diffusion coefficient is small, mass transfer usually takes place near the object. On the basis of this concept the authors mentioned previously derived the diffusion-layer equation. The solution indicates that there are two different regions in the mass-transfer process. One is far away from the sphere where the concentration is uniform. The other is very near the surface, where all the concentration variation takes place.

However, in the present work, a more detailed analysis is made. We find that there are four more regions, and each has a different mass-transfer mechanism. The first three regions which are near the sphere are investigated here, and the treatment of the sixth, the so-called "far-wake" region, is relegated to the Appendix. A sketch of regions 1–5 is shown in Fig. 1.

Region 1: Region far from the sphere

In this region, the presence of the object of a different concentration is not felt by the fluid, and its concentration is uniform and equal to the upstream concentration. Consequently, there is no diffusion. Mass is simply carried along by convection.

Region 2: The diffusion (boundary) layer

This is the region solved by the authors mentioned, and only a brief summary shall be given here. In this region, convection and diffusion normal to the surface are important in mass transfer, while diffusion tangential to the surface is relatively unimportant.

Since diffusion is near the surface, a new coordinate normal to the surface is suggested; that is $Y = (r^* - 1)/\epsilon = y/\epsilon$, where $\epsilon = (Pe/2)^{-\frac{1}{3}}$ and $y = r^* - 1$. When equation (1) is expressed in terms of the new variable, the dominant terms for small ϵ yield [2]:

$$\tfrac{3}{2} Y^2 \cos\theta \frac{\partial \Theta}{\partial Y} - \tfrac{3}{2} Y \sin\theta \frac{\partial \Theta}{\partial \theta} = \frac{\partial^2 \Theta}{\partial Y^2} \quad (2)$$

with the boundary conditions:

1. $\Theta = 0$ at $Y = 0$.
2. $\Theta = 1$ at $Y = \infty$.
3. $\partial\Theta/\partial\theta = 0$ at $\theta = \pi$.

Since the diffusion term in the θ-direction, $\partial^2\Theta/\partial\theta^2$ in equation (1), is dropped, only one boundary condition in this direction can be specified for equation (2), namely condition 3.

Equation (2) can be solved by a similarity transformation. The similarity variable is defined by

$$\eta = Y/g(\theta), \quad (3)$$

where

$$g(\theta) = 3^{\frac{1}{3}} (\tfrac{1}{2} \sin 2\theta - \theta + \pi)^{\frac{1}{3}}/\sin\theta. \quad (4)$$

The solution obtained is

$$\Theta = \frac{1}{\Gamma(\frac{4}{3})} \int_0^{\eta} \exp(-\eta^3)\, d\eta. \quad (5)$$

The local Nusselt number Nu is

$$Nu_{bl}(\theta) = 2 \frac{\partial \Theta}{\partial r^*}\bigg|_{r^*=1} = \frac{2}{\epsilon \Gamma(\frac{4}{3})\, g(\theta)}. \quad (6)$$

As $\theta \to \pi$, the limit of $g(\theta)$ is $2^{\frac{1}{3}}$, which predicts a finite mass-transfer rate at the front. However, as $\theta \to 0$, $g(\theta)$ tends toward infinity. This implies an infinitely thick diffusion layer or no mass transfer at the rear stagnation point and contradicts the assumption that mass transfer occurs close to the surface of the sphere. In addition, if one examines the tangential gradient along the rear axis, one will find that this gradient does not vanish, violating boundary condition 4 of equation (1). Thus, it is the objective of this work to eliminate these discrepancies.

Region 3: Convective region

According to the boundary-layer solution, it is apparent that the diffusion layer expands starting from the front to the back of the sphere. Eventually, this layer will expand sufficiently far into the bulk such that the convective velocity becomes large, and the approximate velocity used in the boundary-layer equation becomes invalid. This can be shown by substituting the boundary-layer solution into equation (1) and examining the behavior of the neglected terms near $\theta = 0$:

$$\begin{aligned}
&\eta^2\Theta' \, [\underline{\tfrac{3}{2}(g^3 \cos\theta + g^2 g' \sin\theta)} \\
&\quad - \eta g^3 (\tfrac{7}{2} g \cos\theta + \tfrac{15}{4} g' \sin\theta)\epsilon + \ldots] \\
&= \underline{\Theta''} + 2\Theta' g(1 - \eta\epsilon g + (\eta\epsilon g)^2 + \ldots)\epsilon \\
&\quad + \left[\eta^2 g'^2 \Theta'' - \eta\Theta' \left(g g'' - 2g'^2 + \frac{\cos\theta}{\sin\theta} g g'\right)\right] \\
&\qquad \times (1 - 2\eta\epsilon g + \ldots)\epsilon^2, \qquad (7)
\end{aligned}$$

where the prime designates the derivative with respect to η or θ.

The underlined terms, the terms of the first order of magnitude as $\epsilon \to 0$, form the boundary-layer equation. However, as $\theta \to 0$, $g(\theta) \to O(1/\theta)$, and equation (7) has the form

$$\left(\frac{1}{\theta^3} - \frac{\epsilon}{\theta^4} + \ldots\right) = 1 + \frac{\epsilon}{\theta} + \frac{\epsilon^2}{\theta^4},$$

with $\eta = O(1)$. The left-hand side represents the convective terms, and the right-hand side, the diffusion terms. It is easy to see that when $\theta = O(\epsilon)$ all the terms in the velocity expansion are of the same order. In other words, as $\theta \to O(\epsilon)$, the velocity components can no longer be approximated by expansions near the surface; consequently, the diffusion-layer approximation breaks down. Acrivos and Goddard [3] have already pointed out that this breakdown occurs as $\theta \to O(\epsilon)$. It is physically reasonable that the complete Stokes velocity must be used because at small angles the diffusion layer becomes thick and most of it is no longer close to the sphere.

At the same time, the above analysis suggests that the appropriate variable for this region can be

$$S = \theta/\epsilon.$$

The results of substituting S into equation (1) are

$$\left(1 - \frac{3}{2r^*} + \frac{1}{2r^{*3}}\right)\frac{\partial \Theta}{\partial r^*} - \left(1 - \frac{3}{4r^*} - \frac{1}{4r^{*3}}\right) \times \frac{S}{r^*}\frac{\partial \Theta}{\partial S} = \frac{\epsilon}{r^{*2}}\left(\frac{\partial^2 \Theta}{\partial S^2} + \frac{1}{S}\frac{\partial \Theta}{\partial S}\right). \quad (8)$$

The diffusion terms on the right of equation (8), being an order of magnitude smaller, can readily be neglected, and we obtain

$$\left(1 - \frac{3}{2r^*} + \frac{1}{2r^{*3}}\right)\frac{\partial \Theta}{\partial r^*} - \left(1 - \frac{3}{4r^*} - \frac{1}{4r^{*3}}\right) \times \frac{S}{r^*}\frac{\partial \Theta}{\partial S} = 0. \quad (9)$$

The solution of equation (9) must match with the asymptotic form of the boundary-layer solution at $\theta = 0$.

Equation (9) shows that in this region convection is the dominant mode of mass transfer, and diffusion is negligible. Consequently, Θ is constant along streamlines, $\Theta = \Theta(\Psi)$, where from Stokes' solution one can define a stream function Ψ appropriate to region 3 as

$$\Psi = \frac{S^2}{3^{\frac{4}{3}}\pi^{\frac{2}{3}}}\left(\frac{1}{r^*} - 3r^* + 2r^{*2}\right). \quad (10)$$

The form of $\Theta(\Psi)$ is determined by matching with the boundary-layer solution. For this purpose, it is only necessary to find the asymptotic boundary-layer solution at $\theta = 0$ in terms of the stream function. As $r^* \to 1$; that is, as r^* asymptotically approaches the boundary layer,

$$\Psi = \theta^2 y^2/(3\pi)^{\frac{2}{3}}\,\epsilon^2. \quad (11)$$

On the other hand, as $\theta \to 0$, from equations (3) and (4) we obtain

$$\eta = \theta y/(3\pi)^{\frac{1}{3}}\,\epsilon. \quad (12)$$

By comparing equations (11) and (12), we find that in the region of matching the relationship between the similarity variable η and the stream function Ψ is

$$\eta = \sqrt{\Psi}. \quad (13)$$

Thus, the solution in region 3 is

$$\Theta(\Psi) = \frac{1}{\Gamma(\frac{4}{3})}\int_0^{\sqrt{\psi}} \exp(-x^3)\,dx, \quad (14)$$

with Ψ defined by equation (10).

Region 4: The rear-axis region

The solution (14) still does not satisfy the boundary condition $\partial\Theta/\partial\theta = 0$ at $\theta = 0$. This can be traced to the neglect of the tangential diffusion terms in equation (8) and indicates the presence of still another region near the rear axis where convection and diffusion in the θ-direction are important, but diffusion in the r-direction is not. At the same time Θ is small, of order $\epsilon^{\frac{1}{3}}$, in this region.

Examination of the terms in equation (1) or (8) neglected in region 3 indicates that the thickness of region 4 is given by $S = O(\sqrt{\epsilon})$. Hence, a suitable variable for this region is

$$s = S/(\sqrt{\epsilon}) = \theta/\epsilon^{\frac{3}{2}},$$

and the corresponding equation is

$$\left(1 - \frac{3}{2r^*} + \frac{1}{2r^{*3}}\right)\frac{\partial \Theta}{\partial r^*} - \left(1 - \frac{3}{4r^*} - \frac{1}{4r^{*3}}\right) \times \frac{s}{r^*}\frac{\partial \Theta}{\partial s} = \frac{1}{r^{*2}}\left(\frac{\partial^2 \Theta}{\partial s^2} + \frac{1}{s}\frac{\partial \Theta}{\partial s}\right). \quad (15)$$

The boundary conditions for equation (15) are

1. $\partial\Theta/\partial s = 0$ at $s = 0$.
2. $\Theta \to (\sqrt{\epsilon})\ s\ (2r^{*2} - 3r^* + 1/r^*)^{\frac{1}{2}}/\Gamma(\frac{4}{3})\ 3^{\frac{2}{3}}\ \pi^{\frac{1}{3}}$ as $s \to \infty$, in order to match with region 3.
3. As $r \to 1$, Θ must match with the solution in region 5 [see equation (19)].

Note that this region, involving the tangential diffusion terms, grows thicker at greater distances from the sphere. Eventually, it blends into a sixth region, the far-wake region, where the depletion of concentration decays, first, as $1/r^*$ along the rear axis and, second, it decays exponentially in $r^*\theta^2$ in the direction normal to the axis.

Region 5: Region near the rear of the sphere

This region is close to the surface at the rear of the sphere. Being close to the surface, the convective velocity is small. Thus, it is probable that diffusion is comparable to convection. As already shown in region 4, on approaching the rear axis, the tangential diffusion becomes important. In addition, analogous to the diffusion-layer concept, the normal diffusion may also be important as the concentration change takes place in a thin region. Order of magnitude comparisons verify these conclusions.

Examination of the radial diffusion terms neglected in regions 3 and 4 show that these terms become important and the treatment of these regions becomes invalid for $y = O(\epsilon)$. Consequently, the appropriate variables in region 5 are

$$Y = y/\epsilon \quad \text{and} \quad S = \theta/\epsilon,$$

and in this region equation (1) reduces for small ϵ to

$$\tfrac{3}{2} Y^2 \frac{\partial \Theta}{\partial Y} - \tfrac{3}{2} Y S \frac{\partial \Theta}{\partial S} = \frac{\partial^2 \Theta}{\partial Y^2} + \frac{\partial^2 \Theta}{\partial S^2} + \frac{1}{S}\frac{\partial \Theta}{\partial S}, \tag{16}$$

and one sees that diffusion and convection in both the r- and θ-directions become important. The boundary conditions for equation (16) are

1. $\Theta = 0 \quad$ at $\quad Y = 0.$
2. $\partial\Theta/\partial S = 0 \quad$ at $\quad S = 0.$
3. $\Theta \to \eta\Theta'(\eta \to 0) = \epsilon Y S/\Gamma(\frac{4}{3})\,(3\pi)^{\frac{1}{3}}$ as $S \to \infty$, in order to match with the boundary-layer solution in region 2.
4. For $Y \to \infty$, an asymptotic solution for equation (16) is found and will be discussed in the following section.

We have carried the analysis far enough to show that radial diffusion is negligible in regions 3 and 4 and that therefore there can be no upstream propagation of effects from these regions, even though radial diffusion is not negligible in region 5 itself. Thus, another way of stating condition 4 is that the radial diffusion term $\partial^2\Theta/\partial Y^2$ must become negligible compared to the other terms in equation (16) as $Y \to \infty$ in order for the solution to match with regions 3 and 4.

In this section we have proceeded logically from one region to another, examining the dominant modes of mass transfer in each region and the limits of validity of the treatment for each region, either by observing whether the boundary conditions are satisfied or by examining the order of magnitude of terms neglected in the equation of convective diffusion. The fact that a limit of validity is found indicates the existence of an adjoining region in which different mechanisms of mass transfer are important. It is not necessary to continue this analysis into the far wake because, as indicated above, radial diffusion is already negligible in regions 3 and 4 and, consequently, there can be no upstream propagation of effects which could affect the mass-transfer rate on the sphere. However, for those who are interested, the far-wake region is discussed in the Appendix.

Since region 5 is in contact with the surface, the normal concentration gradient at the surface will yield the rate of mass transfer in the rear region. Thus, it is essential to obtain the solution of equation (16). Condition 3 on equation (16) indicates that $\Theta = O(\epsilon)$ in region 5. Since $y = O(\epsilon)$, the order of the Nusselt number in this region is $Nu = O(\partial\Theta/\partial y) = O(1)$, in contrast to the front part of the sphere where $Nu = O(1/\epsilon)$, as indicated by equation (6).

SOLUTION FOR THE REGION AT THE REAR OF THE SPHERE

To obtain a solution of equation (16), we might first express the boundary condition 4 of equation (16) more explicitly by obtaining the asymptotic solution of region 5 as $Y \to \infty$. For $Y \to \infty$, at the boundary joining region 5 to regions 3 and 4, $\partial^2\Theta/\partial Y^2$ should approach zero in order to be consistent with equations (9) and (15) of regions 3 and 4. Imposing this condition on equation (16), we obtain

$$\tfrac{3}{2} Y^2 \frac{\partial \Theta}{\partial Y} - \tfrac{3}{2} YS \frac{\partial \Theta}{\partial S} = \frac{\partial^2 \Theta}{\partial S^2} + \frac{1}{S}\frac{\partial \Theta}{\partial S} \quad (17)$$

as $Y \to \infty$. Introduction of the similarity variables

$$\zeta = SY^{\frac{1}{2}} \qquad \text{and} \qquad \Theta = \epsilon Y^{\frac{1}{2}} f(\zeta)$$

reduces equation (17) to

$$\xi f'' + (1 + 3\xi^2/4)f' - 3\xi f/4 = 0, \quad (18)$$

with the boundary conditions

1. $f \to \xi/\Gamma(\frac{4}{3})\ (3\pi)^{\frac{1}{3}}$ as $\xi \to \infty$, to match with the boundary-layer solution.
2. $f' = 0$ at $\xi = 0$.

Equation (18) can readily be solved numerically.

To verify that the solution, $\Theta = \epsilon Y^{\frac{1}{2}} f(\xi)$, agrees with the assumption that $\partial^2\Theta/\partial Y^2 \to 0$ as $Y \to \infty$, we calculate $\partial^2\Theta/\partial Y^2$ and $\partial^2\Theta/\partial S^2$ from this solution with the result

$$\partial^2\Theta/\partial Y^2 = \epsilon(-f + \xi f' + \xi^2 f'')/4Y^{\frac{3}{2}}$$

and

$$\partial^2\Theta/\partial S^2 = \epsilon Y^{\frac{3}{2}} f''.$$

It is obvious that the assumption is not violated. Moreover, the function f provides the boundary condition for region 4 when $r^* \to 1$:

$$\Theta \to \epsilon^{\frac{1}{2}}\sqrt{(r^* - 1)}f[s\sqrt{(r^* - 1)}] \quad \text{as } r^* \to 1 \text{ for region 4.} \quad (19)$$

We can solve equation (18) for f without solving equation (16) in region 5. Thus, region 4 can be solved in connection with region 3 without knowing the solution for all of region 5.

So far, an analytic solution of equation (16) does not seem possible; therefore, a numerical solution is sought. The problem posed was solved by finite-difference methods using successive overrelaxation on a high-speed digital computer and with a mesh size of 0·05. The third and fourth boundary conditions must be applied at finite values of Y and S. At $S = 3$, Θ was set equal to YS plus an asymptotic correction term valid as $S \to \infty$, while at $Y = 3$, $\partial^2\Theta/\partial Y^2$ was set equal to zero.

RESULTS AND DISCUSSION

From the calculated concentration distribution in the rear-stagnation region, one can easily obtain the mass-transfer rate along the surface by calculating the normal gradient at the surface. The results are shown in Fig. 2, where Nu_r is the local Nusselt number and the subscript r denotes the solution for the rear region.

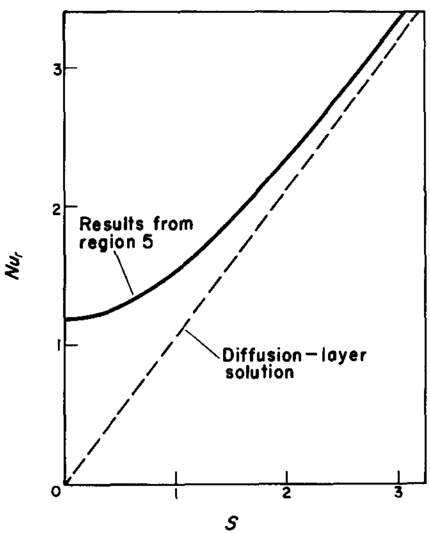

FIG. 2. Local Nusselt number in the region near the rear of the sphere.

Also shown in Fig. 2 is the local Nusselt number Nu_{bl} predicted by the boundary-layer solution. It is clear, as one would expect, that there is a large difference at small angles, and yet the solution for region 5 blends into the boundary-layer solution at larger angles.

At the rear stagnation point, $S = 0$, the local Nusselt number is $Nu_r = 1{\cdot}192$. This is less than the Nusselt number ($Nu = 2$) for the diffusion from a sphere into a stagnant medium. The reason, obviously, is that the convection of depleted solution into the rear region decreases the concentration gradient at the rear.

In order to obtain the mass-transfer rate for the entire sphere, it is necessary to form a composite solution, with the boundary-layer solution as the outer solution and the solution for the rear region as the inner solution. Mathematically, the composite solution for the local Nusselt number can be written as

$$Nu(\theta) = Nu_{bl}(\theta) + Nu_r(S) - 2\theta/\epsilon\Gamma(\tfrac{4}{3})(3\pi)^{\frac{1}{3}}, \tag{20}$$

where the subscript bl denotes the boundary-layer solution. The last term is the inner limit of the boundary-layer solution, which is the same as the outer limit of the solution for region 5.

Acrivos and Goddard have obtained the asymptotic average Nusselt number for $Pe \to \infty$, and $Re \to 0$, as

$$\overline{Nu} = Pe^{\frac{1}{3}}[0{\cdot}9914 + 0{\cdot}922\,Pe^{-\frac{1}{3}} + O(Re) + O(Pe^{-\frac{2}{3}})], \tag{21}$$

and they also have suggested that the contribution of the rear stagnation region to $\overline{Nu}$ is $O(Pe^{-\frac{1}{3}})$, although it actually is of the order $Pe^{-\frac{2}{3}}$. Therefore, if one wants to include the contribution to $\overline{Nu}$ of the rear stagnation region, one should also carry out two more higher order approximations in Acrivos and Goddard's analysis for the diffusion layer.

Even though the contribution of the rear of the sphere to the average Nusselt number is very small, the solution in this region is of interest because it can elucidate one example of the failure of boundary-layer methods at the rear of a bluff object. The solution found here for the rear of a sphere in Stokes flow would also be applicable to other problems at high Péclet numbers, first to the sphere at higher Reynolds numbers where Stokes' velocity profile is not applicable and secondly to other axisymmetric bodies in the absence of eddies behind the bodies, as long as the rear of the body is blunt like the sphere and not pointed. The rear region is very small, and in this region the velocity profile for such an axisymmetric body can be expressed as

$$v_y = by^2 \qquad \text{and} \qquad v_r = -bry,$$

where b is a constant and r denotes the normal distance from the axis of symmetry. In addition one needs the asymptotic form of the mass-transfer rate at the rear of the object as predicted from the diffusion layer on the forward part of the object. Then, by appropriate stretching of the coordinates and concentration, the problem can be reduced to equation (16) and the following boundary conditions, for which the solution has already been given.

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APPENDIX

The Far-Wake Region

The asymptotic solution for region 4 as $r^* \to \infty$ is

$$\Theta \to \sqrt{(\epsilon r^*)}f[s\sqrt{(2r^*/3)}], \tag{22}$$

which, we might note, is very similar to the asymptotic solution in the same region as $r^* \to 1$ [see equation (19)]. The treatment of region 4 ceases to be valid when $\Theta = O(1)$, which occurs when $r^* = O(1/\epsilon)$. This then defines a sixth region, that of the far wake. In this region, the appropriate variables are

$$\mathscr{R} = \epsilon r^* \qquad \text{and} \qquad \mathscr{S} = \theta/\epsilon^2,$$

and equation (1) reduces to

$$\frac{\partial \Theta}{\partial \mathscr{R}} - \frac{\mathscr{S}}{\mathscr{R}}\frac{\partial \Theta}{\partial \mathscr{S}} = \frac{1}{\mathscr{R}^2}\left(\frac{\partial^2 \Theta}{\partial \mathscr{S}^2} + \frac{1}{\mathscr{S}}\frac{\partial \Theta}{\partial \mathscr{S}}\right) \quad (23)$$

with the boundary conditions

1. $\Theta \to 1$ as $\mathscr{S} \to \infty$.
2. $\partial\Theta/\partial\mathscr{S} = 0$ at $\mathscr{S} = 0$.
3. $$\Theta \to \frac{1}{\Gamma(\frac{4}{3})}\int_0^B \exp(-x^3)\,dx + (\sqrt{\mathscr{R}})f[\mathscr{S}\sqrt{(2\mathscr{R}/3)}] - \frac{\mathscr{S}\mathscr{R}\sqrt{(\frac{2}{3})}}{\Gamma(\frac{4}{3})(3\pi)^{\frac{1}{2}}}$$

as $\mathscr{R} \to 0$, where $B = \mathscr{S}\mathscr{R}(\sqrt{2})/3^{\frac{3}{2}}\pi^{\frac{1}{2}}$.

This last boundary condition comes from matching with both region 3 and region 4 and represents the composite expansion for these two regions as $r^* \to \infty$.

The asymptotic solution in region 6 as $\mathscr{R} \to \infty$ is

$$\Theta = 1 - (K/\mathscr{R})\exp(-\mathscr{R}\mathscr{S}^2/4), \quad (24)$$

where the constant K is related to the total rate of mass transfer to the sphere:

$$K = \overline{Nu}/(4Pe)^{\frac{1}{3}} \approx 0{\cdot}6245.$$

Thus, although detailed solutions have not been obtained for regions 4 and 6, one does arrive at a reasonably complete picture of the concentration distribution in the entire flow field to the first approximation as $Pe \to \infty$.

Résumé—On étudie le transport de masse sur une sphère, y compris la région arrière, avec un écoulement de Stokes et des nombres de Péclet élevés. On trouve, par la technique de la perturbation singulière, six régions distinctes avec des mécanismes différents de transport de masse. L'une de ces régions, la couche de diffusion, a déjà été résolue par la méthode de la couche limite. On a donné ici la solution pour une autre région, c'est-à-dire celle à l'arrière de la sphère. On trouve que le nombre de Nusselt local au point d'arrêt aval est 1,192. Afin de prévoir la vitesse de transport de masse en chaque point de la sphère, on peut former une solution mixte à partir des solutions de la couche limite et de la région arrière.

Dans les problèmes de transport de chaleur et de masse, la méthode employée ici complète les méthodes de la couche limite pour prévoir les vitesses de transport de chaleur et de masse à l'arrière d'un objet à symétrie de révolution.

Zusammenfassung—Ex wurde der Stoffübergang an einer Kugel, einschliesslich des rückwärtigen Bereiches in der Stokes-Strömung und bei grossen Péclet-Zahlen untersucht. Nach der Technik der Singular-Strömung wurden sechs verschiedene Bereiche mit unterschiedlichem Stofftransport gefunden. Für einen dieser Bereiche, die Diffusionsschicht, liessen sich bereits Lösungen nach der Grenzschichtmethode angeben. Für einen anderen Bereich, den rückwärtigen Teil einer Kugel, werden Lösungen hier gegeben. Dir örtliche Nusselt-Zahl am rückwärtigen Staupunkt ergab sich zu 1,192. Zur Bestimmung des Stoff transportes überall an der Kugel kann eine zusammengesetzte Lösung aus der Grenzschichtlösung und der Lösung für den rückwärtigen Bereich gebildet werden. In Wärme- und Stoffübergangsproblemen ergänzt die hier verwendete Methode die Grenzschichtmethoden bei der Bestimmung transportierter Wärme oder Masse im rückwärtigen Bereich eines achssymmetrischen Körpers.

Аннотация—Исследован массообмен шара, включая его кормовую стенку, при обтекании стоксовым потоком при больших числах Пекле. Методом теории возмущений было обнаружено шесть участков с различным механизмом массообмена. Решение для одного из этих участков (диффузионного слоя) получено ранее методом пограничного слоя. В данной работе найдено решение для задней стенки. Обнаружено, что локальное число Нуссельта вблизи задней критической точки равно 1,192. Для интенсивности массообмена по всему шару было получено сложное решение на основе решения пограничного слоя и решения для задней стенки. Примененный здесь метод может служить дополнением к методике пограничного слоя при расчете тепло-и массообмена на задней стороне осесимметричного тела.